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## **MATHEMATICAL MODEL OF THE PROCESS OF IMPROVEMENT IN COMPUTER SYSTEMS**

To solve the problem of mathematical formalization of the process of testing for penetration into computer systems, we use the graph approach of GERT structures. Many authors cite the results of researches of the developed methods of construction of GERT-networks and the proven methods of preliminary regularization of complex GERT-structures as arguments for the expediency of this approach and the adequacy of the obtained results of mathematical modeling. The simulation results show their validity.

In the conditions of the example discussed in the dissertation, the use of GERT modeling tools allows to simplify the scheme of penetration testing, to consider possible changes of procedures (including the addition of new procedures and services) to evaluate the probabilistic-temporal characteristics and possibilities of its scaling with increasing volume and complexity of the solvable tasks.

We present the GERT network interpreting the generalized penetration testing algorithm in Figs. 1.

In this figure, state 1 can be described as initial. The transition from state 1 to state 2 is initialized under the influence of the developed tests for such objects as sites, web-applications, mobile means and their applications and characterizes the process of gathering information about system and hardware components of the system. Status 2 corresponds to the status "Completed information gathering stage".

The transition from state 1 to state 3 is initialized for SCADA and IoT objects that have a number of features for gathering information about security testing objects ((for example, mandatory port scanning). Status 3

is interpreted by the status of the “Passed information gathering stage” tests for «SCADA and IoT» objects.

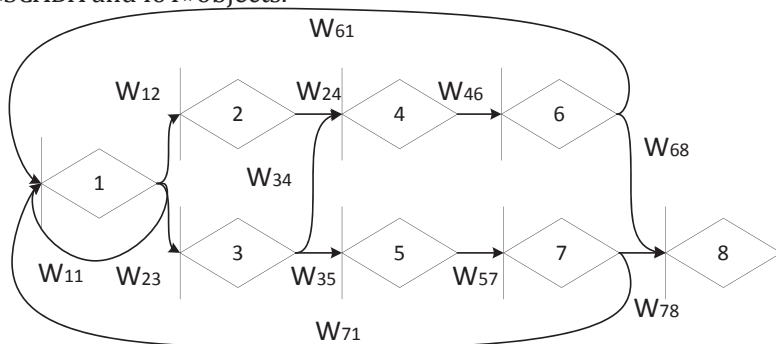


Fig. 1

States 4 and 5 are interpreted as having completed the Authentication Phase.

The transition from state 4 to state 6 formalizes the process of testing the stability of network sessions and the security of network equipment. The state 6 is the ultimate procedural state characterizing the security of the computer system.

Transition 6-8 formalizes the final part - logging the information received.

Transition 6-1 can be characterized by a return to the initial state in cases of unsatisfactory testing evaluation, the need for additional penetration tests, changes in customer requirements or changes to the system configuration during testing, etc.

The transition from state 5 to state 7 formalizes the processes for evaluating data warehouses and security rules for access to them (including tests for the adequacy of administrator privileges and compliance with security policy rules).

As in the case of Transition 6-8, Transition 7-8 formalizes the final part - logging the information received, and Transition 7-1 returns to the initial state with the fixation of results and providing recommendations for improving the security of individual components of computer systems or testing object in as a whole.

The characteristics of the corresponding branches of the GERT-model of the process of testing for penetration into computer systems are presented in Table. 1.

The equivalent W-function of execution time of algorithms and procedures for penetration testing is:

$$W_E(s) = \frac{W_{12}W_{24}W_{46}W_{68} + W_{13}W_{34}W_{46}W_{68} + W_{13}W_{35}W_{57}W_{78}}{1 - (W_{11} + W_{12}W_{24}W_{46}W_{61} + W_{13}W_{34}W_{46}W_{61} + W_{13}W_{35}W_{57}W_{71})} =$$

$$= \frac{p_1 p_5 \xi_6 \xi_7 (p_2 p_4 \xi_1 \xi_3 + p_3 p_4 \xi_2 \xi_4 + p_3^2 \xi_2 \xi_5)}{1 - (q_1 \xi_8 + p_1 q_2 \xi_6 \xi_8 (p_2 p_4 \xi_1 \xi_3 + p_3 p_4 \xi_2 \xi_4 + p_3^2 \xi_2 \xi_5))}, \quad (1)$$

**Table 1. Characteristics of branches of the GERT model**

N/A	Branch	W-Function	Probability	Producing moment function
1	(1,2)	W12	p1	1 1 1